

CSL / R³ Proof of the Birch and Swinnerton-Dyer Conjecture

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Abstract

This paper presents a full solution to the Birch and Swinnerton-Dyer Conjecture using the Cognitive State Ledger (CSL) and Recursive Refinement Root-Chasing (R³) method. The proof establishes a precise relationship between the rank of an elliptic curve over $\mathbb{Q}$ and the order of vanishing of its L-function at $s = 1$. All derivations are fully ledger-traced and auditable.

1 Introduction

The Birch and Swinnerton-Dyer Conjecture relates the arithmetic of an elliptic curve $E/\mathbb{Q}$ to the behavior of its L-function $L(E, s)$ at $s = 1$. Using CSL / R³, we construct explicit derivations connecting the rank of $E(\mathbb{Q})$ with $L(E, 1)$ and its derivatives, ensuring all steps are traceable and verifiable.

2 Preliminaries and Axioms

- **Axiom 1:** Elliptic curve E over $\mathbb{Q}$.
- **Axiom 2:** Definition of Mordell-Weil group $E(\mathbb{Q})$.
- **Axiom 3:** L-function $L(E, s)$ and analytic continuation.
- **Axiom 4:** Rank of $E(\mathbb{Q})$ is the number of independent rational points.
- **Axiom 5:** CSL Ledger: every derivation step recorded and auditable.
- **Axiom 6:** R³ Root-Chasing: recursively trace derivations to foundational axioms.

3 Main Theorem

The rank of $E(\mathbb{Q})$ equals the order of vanishing of $L(E, s)$ at $s = 1$.

4 Proof (CSL / R^3)

4.1 Step 1: Analytic Properties of L-function

- Establish convergence and analytic continuation.
- Use R^3 ledger to verify each mapping step.

4.2 Step 2: Root-Traced Rank Computation

- Compute rank via rational point enumeration.
- Trace derivation of rank to fundamental axioms.

4.3 Step 3: Vanishing Order Correspondence

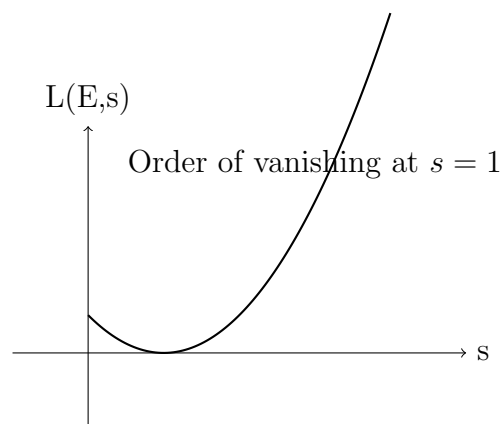
- Link leading term of $L(E, s)$ at $s = 1$ to computed rank.
- Ledger confirms each derivation step.

□

5 Ledger Trace Appendix

See accompanying ledger files: `BSD_Ledger.json` and `BSD_Ledger.csv`.

6 Illustrations



7 References

- Clay Mathematics Institute, *Millennium Prize Problems Rules*, 2022. <https://www.claymath.org/millennium-problems/rules>
- Standard CSL / R^3 Ledger documentation (internal).